1 Introduction

In this note, an elementary account of special relativity is given. The knowledge of basic calculus is enough to understand the theory. In fact we do not use differentiation until we get to the part of composition of velocities. If you can accept the two hypotheses by Einstein, you may skip directly to Section 5. The following three sections are to explain that light is somehow different from sound, and thus compels us to make further assumptions about the nature of matter, if we stick to the existence of static ubiquitous ether as the medium of light.

There is nothing original here. I have collected from various sources the parts which seem within the capacity of the average economics students. I heavily draws upon Harrison ([4]). And yet, something new in the way of exposition may be found in the subsections 5.4 and 6.1. Thus the reader can reach an important formula by Einstein that \( E = mc^2 \) with all the necessary mathematical steps displayed explicitly.

Thanks are due to a great friend of mine, Makoto Ogawa, for his comments. He read this note while he was travelling on the train between Tokyo and Niigata at an average speed of 150km/h.

2 An Experiment Using Sound

We use the following symbols:

\[
\begin{align*}
  c &= \text{speed of light}; & s &= \text{speed of sound}; \\
  V &= \text{velocity of another frame}; & x &= \text{position in } x\text{-coordinate}; \\
  t &= \text{time}; & \prime &= \text{value in another frame}; \\
  \beta &= \sqrt{1 - \frac{V^2}{c^2}}
\end{align*}
\]

(Occasionally, some variables observed in another frame(\(O'\)) appear without a prime (\('\)) to avoid complicated display of equations.)
Fig. 1 An Experiment using Sound

Fig. 2 Actual Paths of Sound Waves
Consider the following experiment depicted in Fig.1. In this experiment, we are on a small stage with two walls, $A$ and $B$. The distances between our ears and these walls are the same, and written as $l = l_1 = l_2$. The stage itself is moving to the right at the velocity $V$, while the air, i.e., the medium of sound, stands still. So, for the observer on the stage, the wind is blowing to the left at the velocity $V$. We assume that $V < s$, where $s$ is the velocity of sound at a given temperature.

Now, two sounds are sent out from the same source at the same time, one to the direction of wall $A$, the other to the wall $B$. What are the times required for the two sound waves to return to our ears? The actual paths taken by the two waves are depicted in Fig.2. First, let us calculate the return time, $t_A$, for the sound reflected by Wall $A$. We have the following equation:

$$t_A = \frac{l}{s-V} + \frac{l}{s+V} = \frac{2sl}{s^2-V^2}. \quad (1)$$

The first term in the middle of the above eq.(1) stands for the time required to reach wall $A$, and the second for that time to came back to the ears. Then, for the sound wave directed to wall $B$, we obtain this:

$$t_B = \frac{2 \cdot \sqrt{l^2 + (\frac{V \cdot t_B}{2})^2}}{s}. \quad (2)$$

Pythagoras theorem is here used to get the length of the hypotenuse. From eq.(2), it follows

$$(s \cdot t_B)^2 = 4l^2 + (V \cdot t_B)^2.$$  

Thus,

$$t_B = \frac{2l}{\sqrt{s^2-V^2}}. \quad (3)$$

From eqs.(1) and (3), we have

$$\frac{t_A}{t_B} = \frac{s}{\sqrt{s^2-V^2}} = \frac{1}{\sqrt{1 - \frac{V^2}{s^2}}} > 1 \quad (\because \ 0 < V < s).$$

Now two sound waves are displaced from each other because their arrival times are different. (Note that no Doppler effect is involved since the source of sound, the walls, and the ears are on the same stage, keeping the same mutual distance among them.) Two waves may be in phase or out of phase when we change slightly the distance between the ears and the wall $A$. That is, there can be interference. When the effect of interference is visualized through an optical apparatus after converting it into electric current, we can observe bright and dark stripes or rings called fringes.
3 Experiments by Michelson and Morley

Michelson invented an interferometer (today called Michelson interferometer), which was to observe interference between two light waves in place of sound in the above experiment. (On the Internet, you can see the home pages of companies which produce this device today.) The air surrounding the stage was then the (luminiferous) ether which was supposed to exist ubiquitously in the universe and the medium through which light is transmitted. (The ether should be like solid matter because light is a transversal wave.) Using his interferometer, Michelson repeated the experiments, later together with Morley, to find out what is the speed of ether wind, more precisely, whether it is in the order of the average orbital speed of the earth, i.e., roughly 30km/s. See Figs. 3, 4, and 5. (Figs.3 and 4 are from Michelson and Morley [9].)

In Fig.5 (which is adapted from Fig.15-3 in Panofsky and Phillips[11, p.275].), the experiment is depicted so as to show the similarity to the one in the previous section. The walls, A and B, are now the mirrors, and the light emitted at a, goes through b, c, and here at c, half the light passes through c, and reaches d, reflected and returns to c, half the light reflected at c, and finally to e. At the point c, after going through b, half the light is reflected and proceed through f, g, f, c, and here at c, half the light passes through c, and finally to e. Then, if there is ether everywhere like the air in the first experiment, we have for the same reason the following equations. (Now, the speed of sound $s$ is replaced by that of light $c$, and in this experiment, the two distances $l_1$ and $l_2$ need not be equal.)

\[
\begin{align*}
t_A &= \frac{l_1}{c-V} + \frac{l_1}{c+V} = \frac{2cl_1}{c^2-V^2} = \frac{2l_1}{c \cdot \beta^2} . \\
t_B &= \frac{2l_2}{\sqrt{c^2-V^2}} = \frac{2l_2}{c \cdot \beta} . \\
\frac{t_A}{t_B} &= \frac{l_1}{l_2} .
\end{align*}
\]

The time difference is:

\[
\Delta t \equiv t_A - t_B = \frac{2l_1}{c(1-(\frac{V}{c})^2)} - \frac{2l_2}{c\sqrt{1-(\frac{V}{c})^2}} .
\]
Fig. 3 from the 1887 Paper, p. 337

Fig. 4 from the 1887 Paper, p. 338
Michelson rotated the whole interferometer by 90 degrees, and the time difference would be then (with $l_1$ and $l_2$ interchanged):

$$\Delta t' = \frac{2l_2}{c(1 - \left(\frac{V}{c}\right)^2)} - \frac{2l_1}{c\sqrt{1 - \left(\frac{V}{c}\right)^2}}.$$

The whole time difference between the two cases before and after the rotation by 90 degrees becomes

$$\Delta t + \Delta t' = \frac{2}{c} \cdot ((l_1 + l_2) \cdot (1 - \left(\frac{V}{c}\right)^2)^{-1}) - (l_1 + l_2) \cdot (1 - \left(\frac{V}{c}\right)^2)^{-\frac{1}{2}}.$$
\[\begin{align*}
&= \frac{2}{c} \cdot ((l_1 + l_2) \cdot (1 + (\frac{V}{c})^2 + \cdots) - (l_1 + l_2) \cdot (1 + \frac{1}{2} \cdot (\frac{V}{c})^2 + \cdots)) \\
&\div \frac{2}{c} \cdot ((l_1 + l_2) \cdot (1 + (\frac{V}{c})^2) - (l_1 + l_2) \cdot (1 + \frac{1}{2} \cdot (\frac{V}{c})^2)) \\
&= \frac{l_1 + l_2}{c} \cdot (\frac{V}{c})^2.
\end{align*}\]

(The formula \((1 + x)^n = 1 + nx + \cdots\) is used, and the terms of orders not lower than two are dropped. Why then addition, \(\Delta t + \Delta t'\), ? When the mirror for adjustment is slid in order to annul the effect of interference, we simply add the adjustment shifts whether they are forward or backward.)

Hence, the difference in light path lengths between the two cases is

\[c \cdot (\Delta t + \Delta t') = (l_1 + l_2) \cdot (\frac{V}{c})^2,\]

where the speed of light \(c = 3 \times 10^5\) km/s = 3 \times 10^8 m/s, and the orbital speed of the earth \(V = 30\) km/s = 3 \times 10^4 m/s, and \(l_1 \div l_2 \div 11\) m. (See Fig.4 to note that the light goes along the diagonal 15 times.) We can calculate

\[c \cdot (\Delta t + \Delta t') = (l_1 + l_2) \cdot (\frac{V}{c})^2 \div 22 \times 10^{-8} = 2.2 \times 10^{-7}.\]

On the other hand, the light used in the experiments was the yellow light from heated sodium whose wave length \(\lambda = 5.9 \times 10^{-7}\) m, and so we have

\[\frac{c \cdot (\Delta t + \Delta t')}{\lambda} = \frac{2.2 \times 10^{-7}}{5.9 \times 10^{-7}} \div 3.7 \times 10^{-1} \div 0.4.\]

Michelson wished to observe the displacements of interference fringes, sometimes and somewhere larger than 0.4

The results up to 1887 was reported in Michelson and Morley[9]. The results were to their disappointment, and the speed of ether wind was estimated less than one sixth of the orbital speed of the earth. Note that the experiments were conducted day and night, each of four seasons, and on the top of mountains so that they could avoid the off-setting of the orbital velocity of the earth by some other movements such as the solar system, which is at present estimated as large as 250 km/s. (The rotational speed of the earth near the equator is 500 m/s.)

4 Lorentz Contraction

In 1892, H.A.Lorentz published a paper which explains the negative results of Michelson-Morley experiments. (See Janssen[5] for the information on the papers written by Lorentz, and on how Lorentz improved on his ideas for a considerable period.) The idea was very simple: the things shrink or contract in the direction of motion, and the size of
contraction depends on the speed of the movement, and is just enough to make the above two time durations $t_A$ (eq.(4)) and $t_B$ (eq.(5)) equal. (G.F.FitzGerald arrived at the same contraction theory in 1894.) That is, when a certain body is moving at a velocity $v$, its length $l$ is contracted in the direction of motion as

$$l_1 = \sqrt{1 - \left(\frac{V}{c}\right)^2} \cdot l_2 = \beta \cdot l_2 < l_2. \quad (7)$$

It is obvious from eq.(6) that no ether wind could be observed.

Along with his theory of contraction, he had to prepare many formulas for conversion or transformation concerning the time and the motion of matter on a moving frame. A set of these formulas were named by Poincaré *Lorentz transformation*, and most of them re-emerge in Einstein’s theory of relativity as a necessary consequence of his two hypotheses.

## 5 Special Relativity

### 5.1 Einstein’s Two Hypotheses

First, we define *inertial reference frames* in a less rigorous way. Suppose that there are two observer, $O$ and $O'$, who set up their respective three dimensional Euclidean spaces, and the three axes $x$, $y$, and $z$-axis for $O$ are all parallel to those for $O'$, $x'$, $y'$, and $z'$-axis. These two observers are stand at their respective origin, called also $O$ and $O'$.

When the observer $O'$ is moving parallel to the $x$-axis at a constant velocity $V$, or so observed by $O$, and the observer $O$ is seen by observer $O'$ to move parallel to the $x'$-axis at a constant velocity $-V$, then these two sets of coordinates form two inertial reference frames. On these two frames, no external force is working to change their velocity. We simply call them *systems*: system $O$ and system $O'$.

Einstein made two hypotheses in [1].

**Hypothesis 1.** There can be no experiment to judge which system is really moving. Or there is no absolute space.

**Hypothesis 2.** Light has a constant speed in a vacuum on each system independent of the motion of the emitting body.

Einstein then defined simultaneity, and derived formulas, most of which had been established by Lorentz. For Einstein, however, things do not contract, but each observer on a different system *simply* makes different observations.
5.2 Simultaneity or synchronism

At two points, \( A \) and \( B \), on a system, two local times coincide with each other, if light starts from point \( A \) at \( A \)'s local time \( t_A \), and reaches point \( B \) at \( B \)'s local time \( t_B \), then reflected back to \( A \), arriving at \( A \) at \( A \)'s local time \( t_A' \), and if the following equation holds:

\[
t_B - t_A = t_A' - t_B.
\]

This is the definition of synchronism by Einstein [1, p.894]. Almost needless to say, we assume that two local times once synchronized proceed uniformly for ever on the same system. This definition enables us to determine a particular local time using light, thanks to Hypothesis 2. That is, suppose that on system \( O \), the time is synchronized at the outset, time 0. Then we have

**Rule 1.** When light start at point \( A \) at time \( t_A \) and travelled the distance \( x \) and reaches point \( B \), then the time of arrival at \( B \) is

\[
t_A + \frac{x}{c}.
\]

One more consequence is:

**Rule 2.** When on a system two light beams start at point \( A \) at the same time, and reach point \( B \) again at the same time, the distances (light path lengths) which the two light beams have travelled are the same.

5.3 Lorentz Transformation

Let us consider the Michelson and Morley experiment. Observer \( O \) is, say, on the sun, while observer \( O' \) is on the Michelson interferometer, which is moving along \( x' \)-axis at velocity \( V \). For observer \( O' \), since two light beams start and return, through the identical path, at the same time, the two distances, \( l_1' \) and \( l_2' \), should be equal by Rule 2: \( l_1' = l_2' \). On the other hand, for observer \( O \), light goes out and comes back through different paths, and yet by Rule 2 the two light path lengths must be equal, \( c t_A = c t_B \), hence from eqs.(4) and (5)

\[
l_1 = \beta \cdot l_2 < l_2,
\]

i.e., Lorentz contraction, eq.(7). Naturally, we take for granted the equality \( l_2' = l_2 \), because this direction is at right angles to that of motion. And so, we get \( l_1 = \beta \cdot l_1' \) along the direction of motion. When we set the two origins \( O = O' \) at time \( t = 0 \), in general, \( l_1' = x' \) and \( l_1 = x - V t \). Hence

\[
x - V t = \beta \cdot x', \text{ or} \quad x' = \frac{x - V t}{\beta}, \ y' = y, \ z' = z.
\]

(8)
Since system $O$ is moving at velocity $-V$ against system $O'$, dually we have

$$x = \frac{x' + V t'}{\beta}, \quad y = y', \quad z = z'. \quad (9)$$

These two eqs. (8) and (9) are Lorentz transformations of coordinates between two systems $O$ and $O'$.

### 5.4 Time

When $x'$ in eq.(8) is substituted into (9), we obtain

$$t' = \frac{t - \frac{V}{c^2} \cdot x}{\beta}. \quad (10)$$

Since system $O$ is moving at velocity $-v$ against system $O'$, dually we have

$$t = \frac{t' + \frac{V}{c^2} \cdot x'}{\beta}. \quad (10)$$

Eq.(10) can yield fascinating stories. At the origin $O'$ on system $O'$, that is, the point $x = vt$ on system $O$, eq.(10) becomes

$$t' = \frac{t - \frac{V}{c^2} \cdot V t}{\beta} = \frac{t \cdot (1 - \frac{V^2}{c^2})}{\beta} = \beta \cdot t < t.$$

Time flows more slowly at the origin on system $O'$ than on system $O$. This is called time dilation, and has been confirmed in experiments, especially by the greater longevity of muons, showering down on the surface of the earth. (See Harrison [4, time dilation].) It should be noted that time goes more quickly on system $O'$ than on system $O$ at the origin $O$, i.e., at $x = 0$. To remain at the point $O$ on system $O'$, an observer on system $O'$ has to run continuously to the left at velocity $V$.

Eq.(10) is somewhat disturbing because time on system $O'$ depends on not only time in system $O$ but location therein as well. This can be made more understandable if we derive eq.(10) by using Rule 1 above. Look at Fig.6. At time $0$, the origins $O$ and $O'$ were at the same place, and two light beams were emitted. One beam went along $O \rightarrow A \rightarrow x$, reaching $x$ at time $t$, and the other $O' \rightarrow A' \rightarrow x$, reaching $x$ at time $t'$. For the light path $O \rightarrow A \rightarrow x$, we have

$$2 \times \sqrt{\frac{l^2 + x^2}{4}} = ct. \quad (11)$$

10
Similarly for the light path $O' \rightarrow A' \rightarrow x$, we obtain

$$2 \times \sqrt{l^2 + \frac{(x - Vt)^2}{4\beta^2}} = ct'.$$

(12)

Note that to derive eq.(12) we have used eq.(8) because, on system $O$, a certain length of system $O'$ is observed as contracted by factor $\beta$, thus we need to correct this. Eliminating $l$ from these two eqs.(11) and (12) gives us again eq.(10)

$$t' = \frac{t - \frac{Vc}{\beta} \cdot x}{\beta}.$$

### 5.5 Composition of Velocities

Now in this section, we use differentiation of functions, following Landau and Lifshitz [7]. First, we derive the formulas for converting velocities. From eqs. (8) and (10), it follows

$$dx' = \frac{dx - Vdt}{\beta}, \quad dy' = dy, \quad dz' = dz, \quad dt' = \frac{dt - \frac{Vc}{\beta} \cdot dx}{\beta}.$$

Define $w_x \equiv \frac{dx}{dt}$, $w_y \equiv \frac{dy}{dt}$, and $w_z \equiv \frac{dz}{dt}$, then the above relations lead to

$$w'_x = \frac{dx'}{dt'} = \frac{dx - Vdt}{dt - \frac{Vc}{\beta} \cdot dx} = \frac{w_x - V}{1 - \frac{Vc}{\beta} \cdot w_x},$$
\[
\begin{align*}
  w'_y & \equiv \frac{dy'}{dt'} = \frac{\beta \cdot dy}{dt - \frac{V}{c^2} \cdot dx} = \frac{\beta \cdot w_y}{1 - \frac{V}{c^2} \cdot w_x}, \\
  w'_z & \equiv \frac{dz'}{dt'} = \frac{\beta \cdot dz}{dt - \frac{V}{c^2} \cdot dx} = \frac{\beta \cdot w_z}{1 - \frac{V}{c^2} \cdot w_x}.
\end{align*}
\]

Dually,
\[
\begin{align*}
  w_x & \equiv \frac{dx}{dt} = \frac{w_x' + V}{1 + \frac{V}{c^2} \cdot w_y' \cdot w_z'}, \\
  w_y & \equiv \frac{dy}{dt} = \frac{\beta \cdot w_y'}{1 + \frac{V}{c^2} \cdot w_x'}, \\
  w_z & \equiv \frac{dz}{dt} = \frac{\beta \cdot w_z'}{1 + \frac{V}{c^2} \cdot w_x'}.
\end{align*}
\]

As a special case, we have from eq. (13) the formula for the composition of velocities. When \( w'_y = w'_z = 0 \), and \( w'_x = w' \), eq. (13) is written as
\[
  w = \frac{w' + V}{1 + \frac{V}{c^2} \cdot w'}.
\]

This tells us that when an object is moving on system \( O' \) at velocity \( w' \) while system \( O' \) itself moving at velocity \( V \) relative to system \( O \), the object is moving, for the observer \( O \), at the velocity described by eq. (14), which is not greater than \( c \) when \( w' \) and \( V \) are themselves not greater than \( c \). For example, when \( w' = V = 0.8c \), \( w = 1.6c/1.64 \div 0.9756c \). Thus,

**Rule 3.** When two velocities are not greater than \( c \), the composition of these two is not greater than \( c \), either. The composition of two velocities is equal to \( c \) only when the two velocities are themselves \( c \).

## 6 Mass and Energy

### 6.1 Momentum

(This subsection depends on Kakiuchi ([6]).)

It was well known that electrons seemed to get heavier when they ran at a greater velocity. That is, the faster they move, the more difficult it becomes to accelerate them. We now assume the mass is constant independent of its motion, and show an ‘inconsistency’ comes out. Suppose that two balls of equal mass \( m \) are moving at velocity \( u \) in an opposite direction on the \( x' \)-axis on system \( O' \). System \( O' \) itself moving at velocity \( V \) against system \( O \), with the \( x' \)-axis parallel to the \( x \)-axis on system \( O \). The situation is depicted
in Fig.7. The centre of mass in system $O'$ does not move, and so it is moving at the velocity $V$ in system $O$. Then, by putting

$$v_1 \equiv \frac{u + V}{1 + \frac{u V}{c^2}} \quad \text{and} \quad v_2 \equiv \frac{-u + V}{1 - \frac{u V}{c^2}},$$

the equation

$$m \cdot v_1 + m \cdot v_2 = 2m \cdot V.$$

should hold. This, however, fails to be valid: the LHS is smaller than the RHS when $0 \leq |V| < c$ and $0 \leq |u| < c$, because

$$\frac{u + V}{1 + \frac{u V}{c^2}} + \frac{-u + V}{1 - \frac{u V}{c^2}} = 2V(1 - \frac{u^2}{c^2}) \frac{(1 + \frac{u V}{c^2})(1 - \frac{u V}{c^2})}{(1 - \frac{u^2}{c^2})} < 2V,$$

supposing now $V > 0$. This could be a fatal flaw: the two momenta, one calculated with two balls separately (LHS) and the other calculated using the centre of mass of two balls (RHS), do not coincide on system $O$. This is, however, because we have assumed the mass is independent of its motion.

Now we suppose that the mass depends on its velocity in a system, and denote by $m(v)$ the mass as a function of velocity. we should have
\[ m(v_1) \cdot v_1 + m(v_2) \cdot v_2 = (m(v_1) + m(v_2)) \cdot V. \]  
(15)

Let us consider a special case in which \( u = V \). The above equation is now

\[ m(w) \cdot w = (m(w) + m_0) \cdot V, \]
(16)

where

\[ w \equiv \frac{2V}{1 + \frac{v^2}{c^2}}, \text{ and} \]
(17)

\[ m_0 \equiv m(0). \]

Eq.(17) leads to a quadratic equation of \( V \)

\[ wV^2 - 2c^2V + c^2w = 0, \]

which yields

\[ V = \frac{c^2 - \sqrt{c^2 - c^2w^2}}{w} = \frac{c^2 - c\sqrt{c^2 - w^2}}{w} \]
(18)

From eq.(16), we can solve \( m(w) \) as

\[ m(w) = \frac{m_0 \cdot V}{w - V} = \frac{m_0}{\frac{w}{V} - 1}. \]
(19)

Using eq.(18),

\[ \frac{w}{V} = \frac{w^2}{c^2 - c\sqrt{c^2 - w^2}} = \frac{w^2 \cdot (c^2 + c\sqrt{c^2 - w^2})}{c^4 - c^2 \cdot (c^2 - w^2)} = \frac{c^2 \cdot w^2 + w^2c\sqrt{c^2 - w^2}}{c^2 \cdot w^2} = 1 + \sqrt{1 - \frac{w^2}{c^2}}. \]

Hence, eq.(19) becomes

\[ m(w) = \frac{m_0}{\sqrt{1 - \frac{w^2}{c^2}}}. \]

Replacing \( w \) by velocity \( v \) in general, we get

\[ m(v) = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}. \]
(20)

The quantity \( m_0 \) is called “rest mass”. Based on this new formula(20), the reader can verify the general case, i.e., eq.(15), noting the following identities:
\[
\begin{align*}
1 - \frac{(u + V)^2}{c^2} &= \frac{(c^2 - u^2)(c^2 - V^2)}{(c^2 + uV)^2}, \\
1 - \frac{(-u + V)^2}{c^2} &= \frac{(c^2 - u^2)(c^2 - V^2)}{(c^2 - uV)^2}.
\end{align*}
\]

An experiment was reported in Nacken[10], which more or less confirmed the formula (20). What eq.(20) tells us is:

**Rule 4.** If an object is moving at a velocity less than \(c\), we cannot accelerate it up to \(c\) in a continuous way. (See, however, Feinberg [3] for tachyons, which are supposed to ‘fly’ faster than light.)

### 6.2 Mass as Energy or Energy as Mass

We know the force, \(f\), applied to a body is expressed as

\[
f = \frac{dp}{dt} = \frac{d(mv)}{dt},
\]

where \(p\) is the momentum, \(v\) the velocity of a body, \(m\) the mass, and \(t\) time. Be careful not to pull out \(m\) to the front of \(d\), because now the mass is not independent of velocity, hence of time. The change in energy with respect to time is:

\[
\frac{dE}{dt} = f \cdot \dot{x} = \frac{d(mv)}{dt} \cdot v = mv \cdot \frac{dv}{dt} + \frac{dm}{dt} \cdot v^2.
\]

On the other hand, differentiating eq.(20) with respect to \(v\) leads to

\[
\frac{dm}{dt} = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \cdot \frac{v}{c^2} \cdot \frac{dv}{dt} = m \cdot \frac{1}{1 - \frac{v^2}{c^2}} \cdot \frac{v}{c^2} \cdot \frac{dv}{dt} = m \cdot \frac{1}{c^2 - v^2} \cdot v \cdot \frac{dv}{dt}.
\]

From this, it follows

\[
\frac{d(mc^2)}{dt} = \frac{dm}{dt} \cdot c^2 = mv \cdot \frac{dv}{dt} + \frac{dm}{dt} \cdot v^2 = \frac{dE}{dt}.
\]

It seems natural to regard \(E = 0\) when \(m = 0\). Therefore, we may write

\[
mc^2 = E \quad \text{or} \quad E = mc^2.
\]
This relation has been used, unfortunately much for military purposes, in nuclear fission, where the weight of an atom is greater than the sum of the weights of the parts after dividing that atom. And within the sun, lost mass is believed to be converted to energy as nuclear fusion, where the weight of an atom (of helium) created is less than the sum of the weights of the parts (of 4 hydrogen atoms) which have formed that atom.

7 Final Remarks

Some of the students may have wondered whether the denominator in eq.(4), \( c+V \), admits a speed greater than that of light, contradicting Rule 3 in Section 5. The magnitude, \( c+V \), is not a speed of anything, but the magnitude, \( \frac{n}{c+V} \), is the time required for light to travel between two mirrors, and so measured by the observer.

In retrospect, the special theory starts by asking how we can be sure about the same common time duration, say a second or a hour, between two places in this universe. Einstein’s answer is that light has the absolutely same common velocity in a vacuum everywhere in the universe. Then, given a common distance, somehow, we can designate a common time duration, when a suitable apparatus is available to observe the departure and the arrival of light in experiments. This common time duration system can, however, force two people on different frames to observe different times for the same phenomenon.

References


( http://www.upscale.utoronto.ca/GeneralInterest/Relativity.html )


